Compound Interest

***Simple Interest*:** If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is $\boldsymbol{I = Prt}$.

When the interest due at the end of a payment period is added to the principal so that the interest computed at the end of the next payment period is based on the new principal amount (old principal plus interest), the interest is said to have been ***compounded*. *Compound interest*** is interest paid on principal and previously earned interest.

Example: A credit union pays 7% per annum compounded quarterly on a certain savings plan. If $900 is deposited in such a plan and the interest is left to accumulate, how much is in the account after 1 year? Hint: Find the simple interest each time the balance is compounded, adding the new interest each time.

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate t compounded n times per year is $\boldsymbol{A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}}$.

Example: Investing $1000 at an annual rate of 9% compounded annually, semiannually, quarterly, monthly, and daily will yield the following amounts after 1 year:

Annual Compounding (n = 1):

Semiannual Compounding (n = 2):

Quarterly Compounding (n = 4):

Monthly Compounding (n = 12):

Daily Compounding (n = 365):

Continuous Compounding

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is $\boldsymbol{A = Pe^{rt}}$.

Example: Find the amount A that results from investing a principle P of $1000 at an annual rate r of 9% compounded continuously for a time t of 1 year.

***Effective Rate of Interest*:** The equivalent annual simple rate of interest that would yield the same amount as compounding after 1 year.

1. Choose any amount to use for principal—the effective rate of interest is the same for any principal.
2. Find the amount resulting by compounding using $A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$ or $A = Pe^{rt}$.
3. Find the interest paid using $I = A - P$.
4. Find the simple interest rate that would result in that amount of interest using $I = Prt$.

Example: Compute the effective rate of interest for 6% compounded continuously.

***Present Value*:** The amount of money that must be invested now in order to end up with a given amount after a certain amount of time.

The present value P of A dollars to be received after t years, assuming a per annum interest rate r is compounded n times per year, is $\boldsymbol{P = A \cdot \left(1 + \frac{r}{n}\right)^{-nt}}$. If the interest is compounded continuously, $\boldsymbol{P = Ae^{-rt}}$.

Example: How much money must be invested now in order to end up with $20,000 in 10 years at

a) 5% compounded quarterly?

b) 3.8% compounded continuously?

Example: A zero-coupon (noninterest-bearing) bond can be redeemed in 9 years for $1200. How much should you be willing to pay for it now if you want a return of

a) 7% compounded monthly?

b) 6% compounded continuously?

Example: What annual rate of interest compounded annually should you seek if you want to double your investment in 7 years?

Example: Find the time required to double or triple an investment if it earns 5% compounded continuously.